Exam. Code : 103202
Subject Code : 1048

B.A./B.Sc. 2nd Semester

MATHEMATICS

Paper—I (Calculus and Differential Equations)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions each from section.

SECTION—A

1. (a) Find the equation of the cubic which has the same asymptotes as the curve $x^2y - xy^2 + xy + y^2 + x - y = 0$ and passes through the points (0, 0), (0, 1) and (1, 0).

   (b) Determine a and b so that the curve $y = ax^3 + 3bx^2$ has a point of inflexion at (–1, –2). 5,5

2. (a) Find the position and nature of the double points on the curve :

   $$2(x^3 + y^3) - 3(3x^2 + y^2) + 12x - 4 = 0.$$

   (b) If ρ_1, ρ_2 are the radii of the curvature at the extremities of a focal chord of a parabola whose semi-latus rectum is l prove that :

   $$(\rho_1)^{-\frac{2}{3}} + (\rho_2)^{-\frac{2}{3}} = (\ell)^{-\frac{2}{3}}.$$ 5,5

3. (a) Trace the curve $y^2 = x(x - a)^2$, $a > 0$.

(b) Evaluate :

$$\int \frac{dx}{a + b\tanh x'}, a^2 \neq b^2.$$ 5,5

4. (a) Obtain a reduction formula for $\int \sec^{2n+1} x dx$. Hence evaluate $\int \sec^5 x dx$.

(b) If $0 < e < 1$, prove that :

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1 - e^2 \sin^2 x}} = 1 + \frac{1^2}{2^2}e^2 + \frac{1^2.3^2}{2^2.4^2}e^4 + \frac{1^2.3^2.5^2}{2^2.4^2.6^2}e^6 + ...$$

5,5

5. (a) Find the area common to the circle $x^2 + y^2 = 4$ and the ellipse $x^2 + 4y^2 = 9$.

(b) Prove that :

$$\int_0^1 \frac{\log(1 + x)}{1 + x^2} dx = \frac{\pi}{8}\log 2.$$ 5,5

SECTION—B

6. (a) Solve :

$$y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0.$$

(b) Solve :

$$p^3 + 8y^2 = 4pxy.$$ 5,5

7. (a) Find the general and singular solution of the equation $xp^2 - (x - a)^2 = 0$.

(b) Prove that the system of confocal and coaxial parabolas $y^2 = 4a(x + a)$ is self orthogonal. 5,5

8. (a) Solve :

$$(D^2 + 1)y = xe^x \sin 2x.$$

(b) Use method of reduction of order to solve :

$$(D^2 + 1)y = \text{cosec } x.$$ 5,5

9. (a) Solve :

$$4x\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0 \text{ in series.}$$

(b) Solve in series :

$$\frac{d^2y}{dx^2} + x^2y = 0.$$ 5,5

10. (a) Solve Legendre's equation :

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0.$$

(b) Solve :

$$(x^2D^2 + 3xD + 1)y = \frac{1}{(1-x)^2}.$$ 5,5